Electric susceptibility

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The **electric susceptibility** χ_e of a dielectric material is a measure of how easily it polarizes in response to an electric field. This, in turn, determines the electric permittivity of the material and thus influences many other phenomena in that medium, from the capacitance of capacitors to the speed of light.

It is defined as the constant of proportionality (which may be a tensor) relating an electric field **E** to the induced dielectric polarization density **P** such that

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E},$$

where ε_0 is the electric permittivity of free space.

The susceptibility of a medium is related to its relative permittivity ε_r by

$$\chi_e \ = \varepsilon_r - 1.$$

So in the case of a vacuum,

$$\chi_e \ = 0.$$

The electric displacement **D** is related to the polarization density **P** by

$$\mathbf{D} \ = \ \varepsilon_0 \mathbf{E} + \mathbf{P} \ = \ \varepsilon_0 (1 + \chi_e) \mathbf{E} \ = \ \varepsilon_r \varepsilon_0 \mathbf{E}.$$

Dispersion and causality

In general, a material cannot polarize instantaneously in response to an applied field, and so the more general formulation as a function of time is

$$\mathbf{P}(t) = \varepsilon_0 \int_{-\infty}^{t} \chi_e(t - t') \mathbf{E}(t') \, dt'.$$

That is, the polarization is a convolution of the electric field at previous times with time-dependent susceptibility given by $\chi_e(\Delta t)$. The upper limit of this integral can be extended to infinity as well if one defines $\chi_e(\Delta t) = 0$ for $\Delta t < 0$. An instantaneous response corresponds to Dirac delta function susceptibility $\chi_e(\Delta t) = \chi_e \delta(\Delta t)$.

It is more convenient in a linear system to take the Fourier transform and write this relationship as a function of frequency. Due to the convolution theorem, the integral becomes a simple product,

$$\mathbf{P}(\omega) = \varepsilon_0 \chi_e(\omega) \mathbf{E}(\omega).$$

This frequency dependence of the susceptibility leads to frequency dependence of the permittivity. The shape of the susceptibilty with respect to frequency characterizes the dispersion properties of the material.

Moreover, the fact that the polarization can only depend on the electric field at previous times (i.e. $\chi_e(\Delta t) = 0$ for $\Delta t < 0$), a consequence of causality, imposes Kramers-Kronig constraints on the susceptibility $\chi_e(0)$.

See also

- Application of tensor theory in physics
- Magnetic susceptibility
- Maxwell's equations
- Permittivity
- Clausius-Mossotti relation

Categories: Electric and magnetic fields in matter | Physical quantity