Pauli equation

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The **Pauli equation** is a Schrödinger equation which describes the time evolution of spin 1/2 particles (eg. electrons). It is the non-relativistic border case of the Dirac equation and can be used where particles are slow enough that relativistic effects can be neglected.

The **Pauli equation** was formulated by Wolfgang Pauli.

Quantum physics

$$\Delta x\, \Delta p \geq \frac{\hbar}{2}$$

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The time dependent linear Pauli equation :

$$(\vec{\sigma}\cdot\vec{p} - c)|\psi\rangle = 0$$

where

$\vec{p}$ is the momentum
c is the speed of light
$\vec{\sigma}$ are the Pauli matrices
$|\psi\rangle := \begin{pmatrix}|\psi_+\rangle \\ |\psi_-\rangle\end{pmatrix}$ is the Pauli spinor

Both spinor components satisfy the Schrödinger-Equation. This means that the system is as to the additional degree of freedom, degenerated.

With an external electromagnetic field the **full Pauli equation** reads:

$$\underbrace{i\hbar\partial_t\vec{\varphi}_\pm = \left(\frac{(\vec{p} - q\cdot\vec{A})^2}{2m} + q\phi\right)\hat{1}\vec{\varphi}_\pm}_{\text{Schrödinger equation}} - \underbrace{\frac{q\hbar}{2m}}_{\text{Stern Gerlach term}}\hat{\vec{\sigma}}\cdot\vec{B}\vec{\varphi}_\pm .$$

where

φ is the scalar electric potenial
A the electromagnetic vector potential
$\vec{\varphi}_\pm$, in Dirac notation $|\psi\rangle := \begin{pmatrix}|\varphi_+\rangle \\ |\varphi_-\rangle\end{pmatrix}$, are the Pauli spinor components
$\hat{\vec{\sigma}}$ are the Pauli matrices
$\vec{B}$ is the external magnetic field
$\hat{1}$ two dimensional Identity matrix

With the Stern Gerlach term it is possible to comprehend the obtaing of spin orientation of atoms with one valence electron e.g. silver atoms which flow through an inhomogenous magntic field.

Analogously, the term is responsible for the splitting of spectral lines (corresponding to energy levels) in a magnetic field as can be viewed in the anomalous Zeeman effect.

Derivation of the Pauli equation

Starting from the Dirac equation for weak electromagnetic interactions :

$$i\hbar\partial_t \begin{pmatrix} \vec{\varphi}_1 \\ \vec{\varphi}_2 \end{pmatrix} = c \begin{pmatrix} \hat{\vec{\sigma}}\vec{\pi}\vec{\varphi}_2 \\ \hat{\vec{\sigma}}\vec{\pi}\vec{\varphi}_1 \end{pmatrix} + q\phi \begin{pmatrix} \vec{\varphi}_1 \\ \vec{\varphi}_2 \end{pmatrix} + mc^2 \begin{pmatrix} \vec{\varphi}_1 \\ -\vec{\varphi}_2 \end{pmatrix}$$

with $\vec{\pi} = \vec{p} - q\vec{A}$

using the following approximatations :

- Simplification of the equation through following ansatz

$$\begin{pmatrix} \vec{\varphi}_1 \\ \vec{\varphi}_2 \end{pmatrix} = e^{-i\frac{mc^2 t}{\hbar}} \begin{pmatrix} \tilde{\vec{\varphi}}_1 \\ \tilde{\vec{\varphi}}_2 \end{pmatrix}$$

- Eliminating the rest energy through an Ansatz with slow time dependence

$$\partial_t \vec{\varphi}_i \ll \frac{mc^2}{\hbar}\vec{\varphi}_i$$

- weak coupling of the electric potential

$$q\phi \ll mc^2$$

Examples

References

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- Claude Cohen-Tannoudji, Bernard Diu, Frank Laloe (2006). *Quantum Mechanics 2*. Wiley, J. ISBN 978-0471569527.

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