Equation

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This article is about equations in mathematics. For the chemistry term, see chemical equation.

An **equation** is a mathematical statement, in symbols, that two things are the same (or equivalent). Equations are written with an equal sign, as in

$$2 + 3 = 5.$$

The equation above is an example of an equality: a proposition which states that two constants are equal. Equalities may be true or false.

Equations are often used to state the equality of two expressions containing one or more variables. Taking the Real numbers we can say, for example, that given any value of x, it is always true that

$$x - x = 0.$$

The equation above is an example of an identity: an equation that is true regardless of the values of any variables that appear within them. The following equation is not an identity:

$$x + 1 = 2.$$

The above equation is false for an infinite number of values of x, and true for only one; the unique root of the equation, $\boldsymbol{x = 1}$. Therefore, if the equation is known to be true, it carries information about the value of x. In general, the values of the variables for which the equation is true are called *solutions*. To solve an equation means to find its solutions.

Many authors reserve the term **equation** for an equality which is not an identity. The distinction between the two concepts can be subtle; for example,

$$(x + 1)^2 = x^2 + 2x + 1$$

is an identity, while

$$(x + 1)^2 = 2x^2 + x + 1$$

is an equation, whose roots are $x = 0$ and $x = 1$. Whether a statement is meant to be an identity or an equation, carrying information about its variables can usually be determined from its context.

Letters from the beginning of the alphabet like a, b, c, ... are often considered constants in the context of the discussion at hand, while letters from end of the alphabet, like x, y, z, are usually considered variables.

Properties

If an equation in algebra is known to be true, the following operations may be used to produce another true equation:

1. Any quantity can be added to both sides.
2. Any quantity can be subtracted from both sides.
3. Any quantity can be multiplied to both sides.
4. Any nonzero quantity can divide both sides.
5. Generally, any function can be applied to both sides. (However, caution must be exercised to ensure that one does not encounter extraneous solutions.)

The algebraic properties (1-4) imply that equality is a congruence relation for a field; in fact, it is essentially the only one.

The most well known system of numbers which allows all of these operations is the real numbers, which is an example of a field. However, if the equation were based on the natural numbers for example, some of these operations (like division and subtraction) may not be valid as negative numbers and non-whole numbers are not allowed. The integers are an example of an

integral domain which does not allow all divisions as, again, whole numbers are needed. However, subtraction is allowed, and is the inverse operator in that system.

If a function that is not injective is applied to both sides of a true equation, then the resulting equation will still be true, but it may be less useful. Formally, one has an implication, not an equivalence, so the solution set may get larger. The functions implied in properties (1), (2), and (4) are always injective, as is (3) if we do not multiply by zero. Some generalized products, such as a dot product, are never injective.

See also

- Inequation
- Inequality
- Linear equation
- Quadratic equation
- Cubic equation
- Quartic equation
- Quintic equation
- Quasi-symmetric equation
- Indeterminate equation
- Differential equation
- Integral equation
- Functional equation
- Diophantine equation
- List of equations
- Theory of equations
- Parametric equation

External links

- Mathematical equation plotter: Plots 2D mathematical equations, computes integrals, and finds solutions online.
- Equation plotter: A web page that can plot general equations, not just functions.
- WZGrapher: A Windows freeware program that plots Cartesian and polar equations, with both integration and differentiation solvers and graphing capabilities.
- Equation Wizard: Automatic algebraic equation solver
- EqWorld — contains information on solutions to many different classes of mathematical equations.
- EquationSolver: A webpage that can solve single equations and linear equation systems.

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